Math 162 Worksheet: Sections 6.2 and 6.3
Volumes and Volumes by Cylindrical Shells



Definitions

Definition of Volume:

Volume Formulas for Common Shapes

- Cube:
- Rectangular Prism:
- Triangular Prism:
- Sphere:
- Cylinder:

Solids of Revolution

What are solids of revolution?:

Practice Problems: Set up each Integral

1. $y = \sin(x),\ y = 0,\ x \in [0, \frac{\pi}{2}]$ about the x-axis
2. $y = \sqrt[3]{x},\ x = 0,\ y = 1$ about the y-axis
3. $y = \sqrt{x},\ x^2 + y^2 = 1,\ x \geq 0,\ y \geq 0$, about the y-axis

Practice Problems: Evaluate each Integral

1. $$\pi \int_0^{\frac{\pi}{2}} \cos(x)\, dx$$
2. $$\pi \int_0^1 y^2\, dy$$
3. $$\pi \int_0^{\frac{\pi}{3}} \tan(x)\, dx$$

Cylindrical Shells Method: Find the Volume

About the y-axis

1. $y = \sqrt{x},\ y = 0,\ x = 1$
2. $y = x^2,\ y = 0,\ x = 1,\ x = 3$
3. $y = x^3,\ x \in [0, 2]$
4. $y = x,\ y = x^2$

About Specific Axes

1. $y = x^2,\ x = \frac{1}{2}y,$ about $x = 3$
2. $y = 5 - 3x,\ x = 1,\ y = 5,$ about $y = 2$
3. $y = \sqrt{x},\ y = 0,$ about $x = 2$

Solutions

- **Definition of Volume:** $V = \lim_{n\to\infty} \sum_{i=1}^{n} A(x_i)\Delta x = \int_a^b A(x)\,dx$
- **Volume Formulas**
  Cube: $V = x^3$
  Rectangular Prism: $V = lwh$
  Triangular Prism: $V = \frac{1}{2}(\text{base})(\text{height}) \times \text{length}$
  Sphere: $V = \frac{4}{3}\pi r^3$
  Cylinder: $V = \pi r^2 h$
- **What are solids of revolution?** Solids generated by revolving a region around a line (the axis of rotation).

$$V = \int_a^b A(x)\,dx \quad \text{(about x-axis)}, \qquad V = \int_c^d A(y)\,dy \quad \text{(about y-axis)}$$

- **Set up the integral**

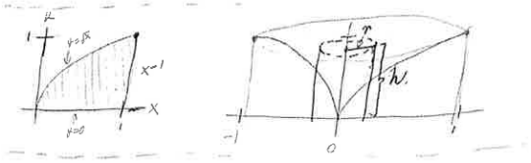
1. $$V = \pi \int_0^{\frac{\pi}{2}} \sin^2(x)\,dx$$
2. $$x = y^3 \Rightarrow V = \pi \int_0^1 (y^3)^2\,dy = \pi \int_0^1 y^6\,dy$$
3. $$x = y^2 \Rightarrow V = \pi \int_0^{.786} 1 - y^2 - y^4\,dy$$

- **Evaluate each integral**

1. $$\pi \int_0^{\frac{\pi}{2}} \cos(x)\,dx = \pi[\sin(x)\big|_0^{\frac{\pi}{2}}] = \pi[\sin(\frac{\pi}{2}) - \sin(0)] = \pi[1-0] = \pi$$
2. $$\pi \int_0^1 y^2\,dy = \pi\left[\frac{y^3}{3}\right]_0^1 = \frac{\pi}{3}$$
3. $$\pi \int_0^{\frac{\pi}{3}} \tan(x)\,dx = \pi \int_0^{\frac{\pi}{3}} \frac{\sin(x)}{\cos(x)}\,dx$$
   $$u = \cos(x), du = -\sin(x), dx = -\frac{du}{sin(x)}$$
   $$\pi \int_0^{\frac{\pi}{3}} -\frac{1}{u}\,dx = \pi[-\ln|\cos(x)|]\big|_0^{\frac{\pi}{3}} =$$
   $$\pi[-\ln|\cos(\frac{\pi}{3})| + \ln|\cos(0)|] = \pi[-\ln|\frac{1}{2}| + \ln|1|] = \pi[-\ln|\frac{1}{2}|] = -\pi\ln[\frac{1}{2}]$$

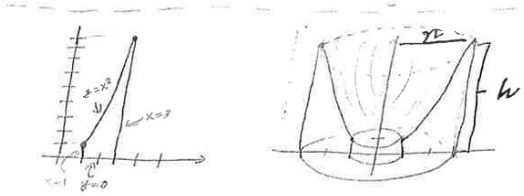
- **Cynlindrical Shells Method**
  About the y-axis

1. $y = \sqrt{x},\ y = 0,\ x = 1$



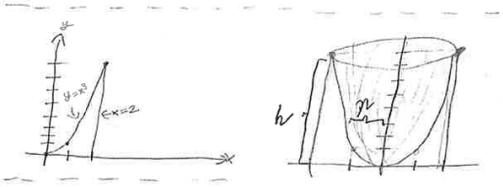
$$r = x,\ h = \sqrt{x},\ V = 2\pi \int_0^1 x\sqrt{x}\,dx = 2\pi \int_0^1 x^{3/2}\,dx = \frac{4\pi}{5}$$

2. $y = x^2,\ y = 0,\ x = 1,\ x = 3$



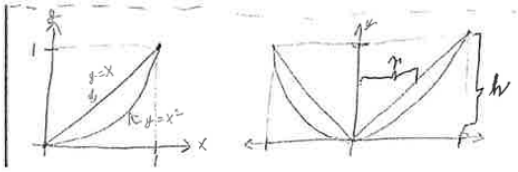
$$r = x,\ h = x^2,\ V = 2\pi \int_1^3 x^3\,dx = 40\pi$$

3. $y = x^3,\ x \in [0, 2]$



$$r = x,\ h = x^3,\ V = 2\pi \int_0^2 x^4\,dx = \frac{64\pi}{5}$$

4. $y = x,\ y = x^2 \Rightarrow h = x - x^2,\ x \in [0, 1]$



$$V = 2\pi \int_0^1 x(x - x^2)\,dx = 2\pi \int_0^1 (x^2 - x^3)\,dx = \frac{\pi}{6}$$

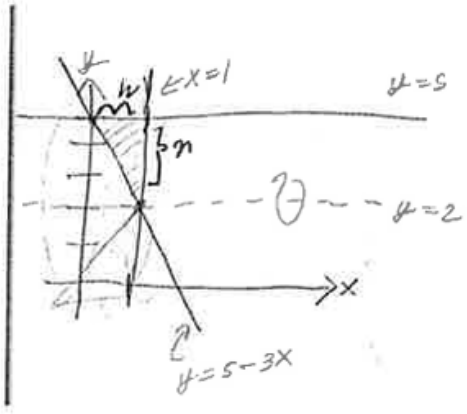
About specific axes

1. $y = x^2,\ x = \frac{1}{2}y,$ about $x = 3$



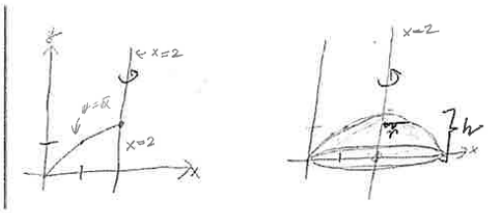
$$r = 3 - x,\ h = 2x - x^2,\ V = 2\pi \int_0^1 (3 - x)(2x - x^2)\, dx = \frac{16\pi}{3}$$

2. $y = 5 - 3x,\ x = 1,\ y = 5,$ about $y = 2$



$$x = \frac{5 - y}{3},\ r = 5 - y,\ h = 2 - \frac{5 - y}{3} - 1,\ V = 2\pi \int_2^5 (5 - y)(2 - \frac{5 - y}{3})\, dy = 12\pi$$

3. $y = \sqrt{x},\ y = 0,$ about $x = 2$



$$r = 2 - x,\ h = \sqrt{x},\ V = 2\pi \int_0^1 (2 - x)\sqrt{x}\, dx = 33.17$$