Math 162 – Integration by Parts Worksheet



What is the formula for Integration by Parts?

Evaluate the integrals:

1. $\int xe^{3x}\,dx$
2. $\int 4x\sin(2x)\,dx$
3. $\int x^2\ln x\,dx$
4. $\int x^2\cos(x)\,dx$
5. $\int x\sin(x)\cos(x)\,dx$
6. $\int_0^1 \tan^{-1}(x)\,dx$

Solutions

Formula for Integration by Parts:

$$\int u\,dv = uv - \int v\,du$$

1. Let $u = x$, $dv = e^{3x}dx$
   Then $du = dx$, $v = \frac{1}{3}e^{3x}$

$$\int xe^{3x}\,dx = \frac{1}{3}xe^{3x} - \int \frac{1}{3}e^{3x}dx = \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$$

2. Let $u = 4x$, $dv = \sin(2x)dx$
   Then $du = 4dx$, $v = -\frac{1}{2}\cos(2x)$

$$\int 4x\sin(2x)\,dx = -2x\cos(2x) + 2\int \cos(2x)\,dx = -2x\cos(2x) + \sin(2x) + C$$

3. Let $u = \ln x$, $dv = x^2dx$
   Then $du = \frac{1}{x}dx$, $v = \frac{x^3}{3}$

$$\int x^2\ln x\,dx = \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\cdot\frac{1}{x}dx = \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2dx = \frac{x^3}{3}\ln x - \frac{x^3}{9} + C$$

4. Let $u = x^2$, $dv = \cos x\,dx$
   Then $du = 2x\,dx$, $v = \sin x$

$$\int x^2\cos x\,dx = x^2\sin x - \int 2x\sin x\,dx$$

   Use integration by parts again on $\int 2x\sin x\,dx$:
   Let $u = 2x$, $dv = \sin x\,dx$
   Then $du = 2dx$, $v = -\cos x$

$$\int 2x\sin x\,dx = -2x\cos x + 2\int \cos x\,dx = -2x\cos x + 2\sin x$$

   So,

$$\int x^2\cos x\,dx = x^2\sin x + 2x\cos x - 2\sin x + C$$

5. Use identity: $\sin x \cos x = \frac{1}{2}\sin(2x)$

$$\int x \sin x \cos x \, dx = \frac{1}{2}\int x \sin(2x)\, dx$$

Let $u = x$, $dv = \sin(2x)\, dx$
Then $du = dx$, $v = -\frac{1}{2}\cos(2x)$

$$\frac{1}{2}\left(-\frac{1}{2}x\cos(2x) + \frac{1}{2}\int \cos(2x)\, dx\right) = -\frac{1}{4}x\cos(2x) + \frac{1}{4}\cdot\frac{1}{2}\sin(2x) + C$$

$$= -\frac{1}{4}x\cos(2x) + \frac{1}{8}\sin(2x) + C$$

6.

$\int_0^1 \tan^{-1}(x)\, dx$
Let $u = \tan^{-1}(x)$, $dv = dx$
Then $du = \frac{1}{1+x^2}dx$, $v = x$

$$\int \tan^{-1}(x)\, dx = x\tan^{-1}(x) - \int \frac{x}{1+x^2}dx$$

Use u-substitution: Let $u = 1 + x^2 \Rightarrow du = 2xdx \Rightarrow xdx = \frac{1}{2}du$

$$\int \frac{x}{1+x^2}dx = \frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\ln|u| = \frac{1}{2}\ln(1+x^2)$$

$$\int_0^1 \tan^{-1}(x)\, dx = \left(x\tan^{-1}(x) - \frac{1}{2}\ln(1+x^2)\right)\Big|_0^1$$

$$= (1)\tan^{-1}(1) - \frac{1}{2}\ln(1+(0)^2) = \frac{\pi}{4} - \frac{1}{2}\ln|2|$$