STEM Success Center

Math 105/115 - Difference Quotient Worksheet

For each equation, evaluate for $\frac{f(x+h)-f(x)}{h}$.

1) $f(x) = 4 - x$

2) $f(x) = x^2$

3) $f(x) = 2x^2 + 8$

4) $f(x) = -x^2 + 5x + 1$

Solutions:

1) Find $f(x+h)$ by replacing x in $f(x)$ with $x+h$:

$$f(x+h) = 4-(x+h)$$

Plug $f(x+h)$ and $f(x)$ into $\frac{f(x+h)-f(x)}{h}$:

$$\frac{4-(x+h)-(4-x)}{h}$$

Distribute the negative into $(x+h)$ and $(4-x)$:

$$\frac{4-x-h-4+x}{h}$$

Cancel out like terms:

$$\frac{4-\cancel{x}-h-4+\cancel{x}}{h}$$

$$= -\frac{h}{h}$$

$$= -1$$

2) Find $f(x+h)$ by replacing x in $f(x)$ with $x+h$:

$$f(x+h) = (x+h)^2$$

Plug $f(x+h)$ and $f(x)$ into $\frac{f(x+h)-f(x)}{h}$:

$$\frac{(x+h)^2-x^2}{h}$$

Expand and FOIL $(x+h)^2$:

$$\frac{(x+h)(x+h)-x^2}{h}$$

$$\frac{x^2+2hx+h^2-x^2}{h}$$

Cancel out like terms:

$$\frac{\cancel{x^2}+2hx+h^2-\cancel{x^2}}{h}$$

$$\frac{2hx + h^2}{h}$$

Factor out h from the numerator and simplify:

$$\frac{h(2x + h)}{h}$$

$$= 2x + h$$

3) Find $f(x + h)$ by replacing x in $f(x)$ with $x + h$:

$$f(x + h) = 2(x + h)^2 + 8$$

Plug $f(x + h)$ and $f(x)$ into $\frac{f(x+h)-f(x)}{h}$:

$$\frac{2(x + h)^2 + 8 - (2x^2 + 8)}{h}$$

FOIL $(x + h)^2$ and distribute the negative into $(2x^2 + 8)$:

$$\frac{2(x + h)(x + h) + 8 - (2x^2 + 8)}{h}$$

$$= \frac{2(x^2 + xh + xh + h^2) + 8 - (2x^2 + 8)}{h}$$

$$= \frac{2(x^2 + 2xh + h^2) + 8 - (2x^2 + 8)}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 + 8 - (2x^2 + 8)}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 + 8 - 2x^2 - 8}{h}$$

Cancel out like terms from the numerator:

$$= \frac{\cancel{2x^2} + 4xh + 2h^2 + \cancel{8} - \cancel{2x^2} - \cancel{8}}{h}$$

$$= \frac{4xh + 2h^2}{h}$$

Factor out h and simplify:

$$= \frac{h(4x + 2h)}{h}$$

$$= 4x + 2h$$

4) Find $f(x + h)$ by replacing x in $f(x)$ with $x + h$:

$$f(x + h) = -(x + h)^2 + 5(x + h) + 1$$

Plug $f(x + h)$ and $f(x)$ into $\frac{f(x+h)-f(x)}{h}$:

$$\frac{-(x + h)^2 + 5(x + h) + 1 - (-x^2 + 5x + 1)}{h}$$

Expand and FOIL $(x + h)^2$:

$$\frac{-(x + h)(x + h) + 5(x + h) + 1 - (-x^2 + 5x + 1)}{h}$$

$$= \frac{-(x^2 + xh + xh + h^2) + 5(x + h) + 1 - (-x^2 + 5x + 1)}{h}$$

$$= \frac{-(x^2 + 2xh + h^2) + 5(x + h) + 1 - (-x^2 + 5x + 1)}{h}$$

Distribute 5 into $(x + h)$:

$$\frac{-(x^2 + 2xh + h^2) + 5x + 5h + 1 - (-x^2 + 5x + 1)}{h}$$

Distribute negatives into $(x^2 + 2xh + h^2)$ and $(-x^2 + 5x + 1)$:

$$\frac{-x^2 - 2xh - h^2 + 5x + 5h + 1 + x^2 - 5x - 1}{h}$$

Cancel out like terms:

$$\frac{\cancel{-x^2} - 2xh - h^2 + \cancel{5x} + 5h + \cancel{1} + \cancel{x^2} - \cancel{5x} - \cancel{1}}{h}$$

$$= \frac{-2xh - h^2 + 5h}{h}$$

Factor out h and simplify:

$$\frac{h(-2x-h+5)}{h}$$

$$=-2x-h+5$$