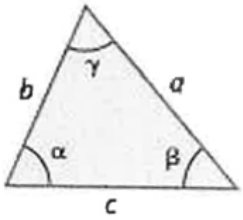
Math 125: Additional Topics in Trigonometry Worksheet

Solve the Triangle



1. Given: $\alpha = 41°$, $\beta = 33°$, $c = 21$ cm
   Find γ, a, and b.
2. Given: $\beta = 83°$, $\gamma = 77°$, $c = 25$ miles
   Find α, a, and b.

Find Area of Triangle

1. $a = 542$ yd, $b = 167$ yd, $\alpha = 90°$
2. $\alpha = 41°$, $\beta = 113°$, $c = 9.5$ m

Vector Magnitude and Operations

Find the magnitude of each vector:

1. $\vec{v} = (4, 3)$
2. $\vec{v} = 2\vec{i} - \vec{j}$
3. If $\vec{u} = (-1, 2), \vec{v} = (3, -2), \vec{w} = (0, -2)$, find:
   (A) $\vec{u} + \vec{v}$
   (B) $\vec{u} - \vec{v}$
   (C) $2\vec{u} - \vec{v} + 3\vec{w}$

Polar to Rectangular Coordinates

Convert each polar coordinate to a rectangular coordinate:

1. $(r, \theta) = \left(7, \frac{2\pi}{3}\right)$
2. $(r, \theta) = (-9.028, -0.663)$

Rectangular to Polar Coordinates

Convert each rectangular coordinate to a polar coordinate:

1. $(x, y) = (6.9, 4.7)$
2. $(x, y) = (-8.33, 4.29)$

Rectangular to Polar Equations

Convert each rectangular equation to a polar equation:

1. $2xy = 1$
2. $6x - x^2 = y^2$

Polar to Rectangular Equations

Convert each polar equation into a rectangular equation:

1. $r = 8\cos\theta$
2. $r(2\cos\theta + \sin\theta) = 4$

Solutions

Solve the Triangle

1.

$$\gamma = 180^\circ - 41^\circ - 33^\circ = 106^\circ$$
$$\frac{\sin 41^\circ}{a} = \frac{\sin 106^\circ}{21} \Rightarrow a = \frac{21 \sin 41^\circ}{\sin 106^\circ} \approx 14.33 \text{ cm}$$
$$\frac{\sin 33^\circ}{b} = \frac{\sin 106^\circ}{21} \Rightarrow b = \frac{21 \sin 33^\circ}{\sin 106^\circ} \approx 11.89 \text{ cm}$$

2.

$$\alpha = 180^\circ - 83^\circ - 77^\circ = 20^\circ$$
$$a = \frac{25 \sin 20^\circ}{\sin 77^\circ} \approx 8.77 \text{ mi}$$
$$b = \frac{25 \sin 83^\circ}{\sin 77^\circ} \approx 25.46 \text{ mi}$$

Find Area of Triangle

1.

$$\frac{\sin 90^\circ}{542} = \frac{\sin b}{21} \Rightarrow b \approx 17.946$$
$$\Rightarrow \gamma \approx 180^\circ - 90^\circ - 17.946^\circ \approx 72.054^\circ$$
$$\Rightarrow Area = \frac{1}{2}(542)(167)\sin(72.054) \approx 43055.16 \text{ yards}^2$$

2.

$$\gamma = 180^\circ - 41^\circ - 113^\circ = 26^\circ$$
$$\frac{\sin 113^\circ}{b} = \frac{\sin 26^\circ}{9.5} \Rightarrow b \approx 19.95 \text{ m}$$
$$h = 19.95 \sin 41^\circ \approx 13.09 \text{ m}$$
$$\text{Area} = \frac{1}{2}(9.5)(13.09) \approx 62.16 \text{ m}^2$$

Vector Magnitude and Operations

1.

$$|\vec{v}| = \sqrt{(-4)^2 + 3^2} = \sqrt{25} = 5$$

2.

$$|\vec{v}| = \sqrt{2^2 + (-1)^2} = \sqrt{5} \approx 2.24$$

3. (A) $\vec{u} + \vec{v} = (-1 + 3, 2 + (-2)) = (2, 0)$
   (B) $\vec{u} - \vec{v} = (-1 - 3, 2 - (-2)) = (-4, 4)$
   (C) $2\vec{u} - \vec{v} + 3\vec{w} = (-2, 4) - (3, -2) + (0, -6) = (-5, 0)$

Polar to Rectangular Coordinates

1.
$$x = 7\cos\left(\frac{2\pi}{3}\right) = -3.5, \quad y = 7\sin\left(\frac{2\pi}{3}\right) \approx 6.06 \Rightarrow (-3.5, 6.06)$$

2.
$$x = -9.028\cos(-0.663) \approx -7.11, \quad y = -9.028\sin(-0.663) \approx 5.55 \Rightarrow (-7.11, 5.55)$$

Rectangular to Polar Coordinates

1.
$$r = \sqrt{6.9^2 + 4.7^2} \approx 8.34, \quad \theta = \arctan\left(\frac{4.7}{6.9}\right) \approx 34.7^\circ \Rightarrow (8.34, 34.7)$$

2.
$$r = \sqrt{(-8.33)^2 + 4.29^2} \approx 9.36, \quad \theta = \arctan\left(\frac{4.29}{-8.33}\right) \approx -27.4^\circ \Rightarrow (9.36, -27.4)$$

Rectangular to Polar Equations

1. $2xy = 1$ becomes:
$$2r^2\cos\theta\sin\theta = 1 \Rightarrow r^2 = \frac{1}{2\cos\theta\sin\theta} \Rightarrow r = \pm\sqrt{\frac{1}{2\cos\theta\sin\theta}}$$

2. $6x - x^2 = y^2$ becomes: $6(r\cos\theta) - (r\cos\theta)^2 = (r\sin\theta)^2 \Rightarrow 6r\cos\theta - r^2\cos^2\theta = r^2\sin^2\theta$

Polar to Rectangular Equations

1.
$$r = 8\cos\theta \Rightarrow x^2 + y^2 = 8x \Rightarrow (x - 4)^2 + y^2 = 16$$

2.
$$2x + y = 4 \Rightarrow y = -2x + 4$$