California State University
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MATH 160

1 Implicit Differentiation

Functions seen so far can be described by expressing one variable explicitly in terms of another variable.
For Example:

$$f(x) = \sqrt{x^2 - 1}, \qquad g(x) = \sin(x), \qquad h(x) = \frac{x^2 + 1}{x^3 + 5}$$

Let's say we wanted to differentiate the function: $x^2 + y^2 = 25$, we are used to differentiating in terms of one variable, but now we have two variables. To keep things simple, since we are dealing with functions in respect of one variable, we will treat the second variable in respect to the other variable, but we will need to apply chain rule on those terms.

2 Example

Find $\dfrac{dy}{dx}$ by implicit differentiation:

$$x^3 + y^3 = 6xy$$

1. Differentiate the left side of the equation:

   $$3x^2 + 3y^2 \cdot \frac{dy}{dx}$$

   Note that the term y^3 must be applied by the chain rule.

2. Differentiate the right side of the equation:

   $$6y + 6x\frac{dy}{dx}$$

   Note that the term $6xy$ must be applied by the product rule.
   Also note that the subterm y must be applied by chain rule within that product rule.

3. Solve for $\frac{dy}{dx}$:

$$3x^2 + 3y^2\frac{dy}{dx} = 6y + 6x\frac{dy}{dx}$$

$$3y^2\frac{dy}{dx} - 6x\frac{dy}{dx} = 6y - 3x^2$$

$$\frac{dy}{dx}\Big(3y^2 - 6x\Big) = 6y - 3x^2$$

$$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$$

$$\frac{dy}{dx} = \frac{2y - x^2}{y^2 - 2x}$$