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# The Laguage of Sets

## What is a set?

**• Notation**

If $S$ is a set, the notation $x \in S$ means that $x$ is an element of $S$. The notation $x \notin S$ means that $x$ is not an element of $S$. A set may be specified using the **set-roster notation** by writing all of its elements between braces. For example, $\{1, 2, 3\}$ denotes the set whose elements are 1, 2, and 3. A variation of the notation is sometimes used to describe a very large set, as when we write $\{1, 2, 3, \ldots, 100\}$ to refer to the set of all integers from 1 to 100. A similar notation can also describe an infinite set, as when we write $\{1, 2, 3, \ldots\}$ to refer to the set of all positive integers. (The symbol $\ldots$ is called an **ellipsis** and is read "and so forth.")

- A set is essentially a group of elements
- We use {.....} to denote a set and the elements go inbetween separated by commas

| Symbol | Set |
|---|---|
| R | set of all real numbers |
| Z | set of all integers |
| Q | set of all rational numbers, or quotients of integers |

- We also use symbols to identify certain sets
- R examples(√2,11, ⅓, -4 )
- Z examples(-3,-2,-1,0,1,2,3)
- Q examples(-5,-½,3.5,⅓)

**• Set-Builder Notation**

Let $S$ denote a set and let $P(x)$ be a property that elements of $S$ may or may not satisfy. We may define a new set to be **the set of all elements $x$ in $S$ such that $P(x)$ is true**. We denote this set as follows:

$$\{x \in S \mid P(x)\}$$

the set of all — such that

- For example what if P(x) says that all the numbers must be positive
- So x ∈ Z | P(x) would means that the set of all integers such that they are positive

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**• Definition**

If $A$ and $B$ are sets, then $A$ is called a **subset** of $B$, written $A \subseteq B$, if, and only if, every element of $A$ is also an element of $B$.

Symbolically:

$$A \subseteq B \quad \text{means that} \quad \text{For all elements } x, \text{ if } x \in A \text{ then } x \in B.$$

The phrases $A$ *is contained in* $B$ and $B$ *contains* $A$ are alternative ways of saying that $A$ is a subset of $B$.

⊆ = subset

- For example A = {1,2,3,4} and B = {1,2,3,4}
- Is B ⊆ A …. Yes
- Is A ⊆ B… Yes

- For example A = {1,2,3,4} and B = {1,2}
- Is B ⊆ A… yes
- Is A ⊆ B… no because it has elements that are not in B (3 and 4)

- If A ⊆ B and B ⊆ A then A=B
    - If two sets are subsets of each other, we know they have the same elements
- Z ⊆ Q ⊆ R

**• Definition**

Let $A$ and $B$ be sets. $A$ is a **proper subset** of $B$ if, and only if, every element of $A$ is in $B$ but there is at least one element of $B$ that is not in $A$.

- For example A = {1,2,3,4} and B = {1,2,3,4}
- Is B a proper subset of A… No because there must be one element of A that is not in B

- For example A = {1,2,3,4} and B = {1,2}
- Is B a proper subset of A.. Yes because its elements are in A but at least one element of A is not in B(3,4)