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# Discrete Mathematics (Math 240): Possibilities and Permutations

- **Sample space**
  - The set of all possible outcomes
- **Event**
  - A subset of a sample space where something occurs

**How to solve probabilities?**

**Equally Likely Probability Formula**

If $S$ is a finite sample space in which all outcomes are equally likely and $E$ is an event in $S$, then the **probability of $E$**, denoted $P(E)$, is

$$P(E) = \frac{\text{the number of outcomes in } E}{\text{the total number of outcomes in } S}.$$

- With this rule we can always find the probability
- In order to use this formula we must know total number of outcomes(S) and the number of desirable outcomes(E)
- For example if I picked a number 1-100 but I wanted to know the probability of picking a single digit number
  - Our E would be a size of 9 for the 9 single digit numbers
  - Our S would be a size of 100 for all of the numers 1-100
  - 9/100 = 9% probability

**What if there are multiple steps in order to find the probability?**

- MULTIPLE STEPS = MULTIPLY

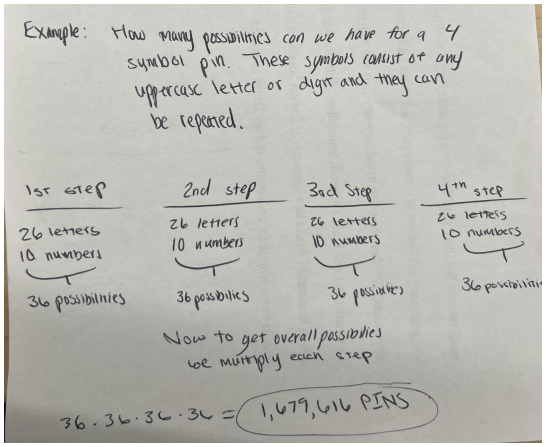
**Theorem 9.2.1 The Multiplication Rule**

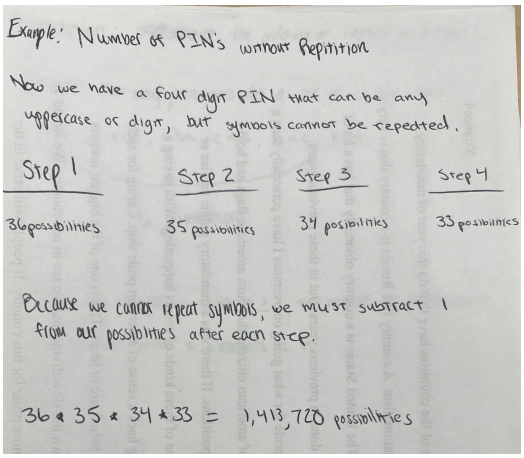
If an operation consists of $k$ steps and

the first step can be performed in $n_1$ ways,
the second step can be performed in $n_2$ ways *[regardless of how the first step was performed]*,
⋮
the $k$th step can be performed in $n_k$ ways *[regardless of how the preceding steps were performed]*,

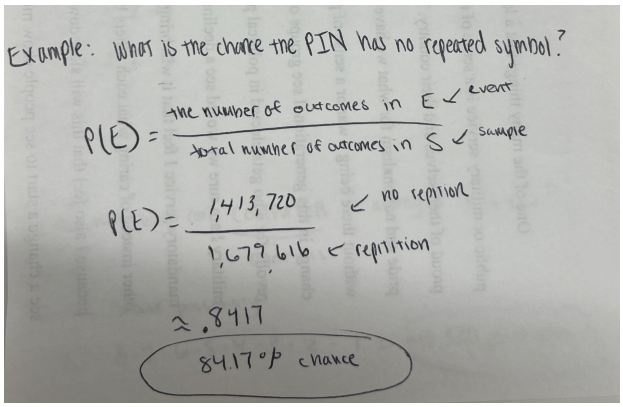
then the entire operation can be performed in $n_1 n_2 \cdots n_k$ ways.

- We must calculate each step and multiply them together in order to get the overall probability or possibilities



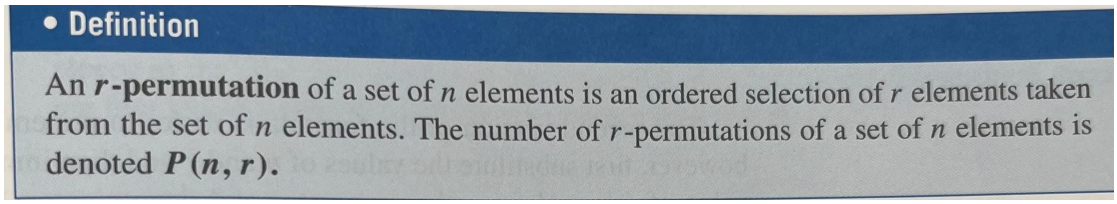

- In left problem we have a 4 symbol pin with 36 possibilities for each symbol
- We then multiply to get the total number of outcomes
- In the right problem we have the same prompt but we cannot repeat symbols
- This means after each step we must subtract 1 possibility as symbol is taken
- We then get the total number of outcomes for no repetition through multiplication




- Remember P = E/S
- Our E is the number of outcomes for a non-repeated 4 symbol pin (1,413,720)
- Our S is the total number of outcomes for any 4 symbol pin(1,679,616)
- We then use P = E/S in order to get the probability of non-repitiion

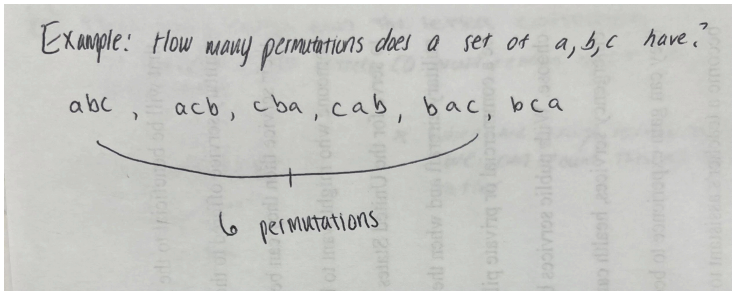
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**What are Permutations and Combinations?**

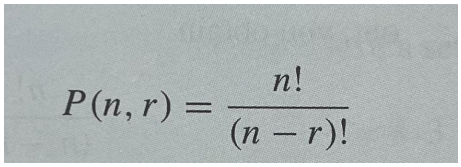


**Definition**

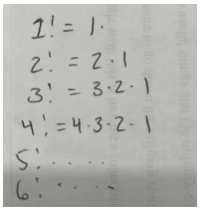
An ***r*-permutation** of a set of $n$ elements is an ordered selection of $r$ elements taken from the set of $n$ elements. The number of $r$-permutations of a set of $n$ elements is denoted $P(n, r)$.

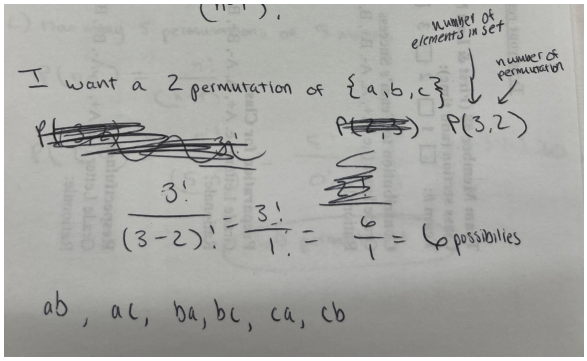



- In simple form, it is all of the possible combinations of a particular set of elements



$$P(n, r) = \frac{n!}{(n-r)!}$$

- R = the desired number of permutations
- N = the total number of elements in the set
- ! = factorial… this means…
  - 

- R value is 2 because based on prompt
- N value is 3 because we have a set of a, b, c
- We plug in and end up with 6 possibilities of 2 permutations

**Combinations**

**• Definition**

Let $n$ and $r$ be nonnegative integers with $r \leq n$. An ***r*-combination** of a set of $n$ elements is a subset of $r$ of the $n$ elements. As indicated in Section 5.1, the symbol

$$\binom{n}{r},$$

which is read "$n$ choose $r$," denotes the number of subsets of size $r$ ($r$-combinations) that can be chosen from a set of $n$ elements.

- **N** represents the number of elements in the set
- **R** represents the amount of elements in each combination

**Theorem 9.5.1**

The number of subsets of size $r$ (or $r$-combinations) that can be chosen from a set of $n$ elements, $\binom{n}{r}$, is given by the formula

$$\binom{n}{r} = \frac{P(n,r)}{r!} \quad \text{first version}$$

or, equivalently,

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} \quad \text{second version}$$

where $n$ and $r$ are nonnegative integers with $r \leq n$.

- If n = 4 and r = 3 this means we have a set of 4 elements and we want to see all of the different ways we can combine 3 of these elements out of the 4 elements we have WITHOUT REPEATING
- This would result in 4 combinations after plugging into either formula

WITHOUT FORUMULA

- For example if I have a set of {a,b,c} and I want all of the combinations of 2
- Then {a,b}, {a,c}, {b,c} would be our 3 combinations
- We are not allowed to repeat like permutations, this is why we dont include {b,a} or {c,a} or {c,b}... these are just different orders but same combinations