Math 105/115 - Graphing Polynomials

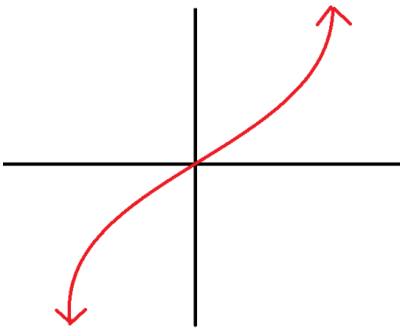
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1 The Leading Coefficient Test

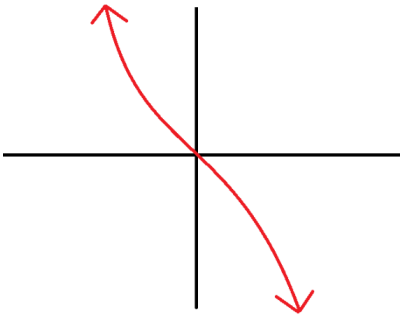
As x moves without bound to the left or to the right, the graph of the polynomial function $f(x) = a_n x^n + ... + a_1 x + a_0$ eventually rises or falls in the following manner:

1) When n is odd:

   a) If the leading coefficient is positive ($a_n > 0$), the graph falls to the left and rises to the right.

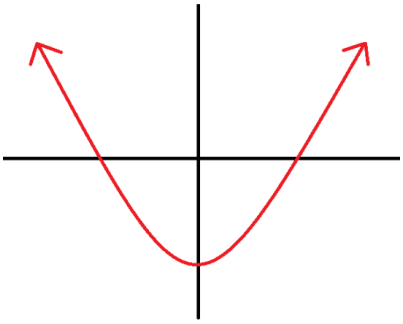


b) If the leading coefficient is negative ($a_n < 0$), the graph rises to the left and falls to the right.

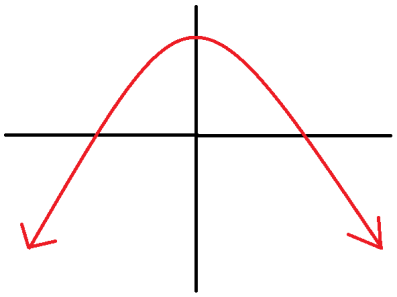


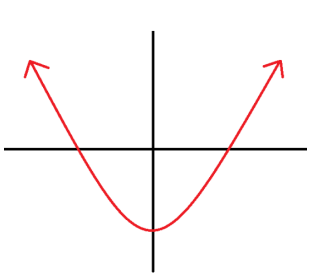
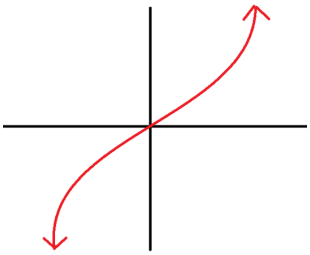
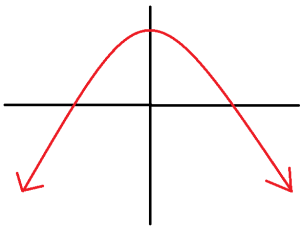
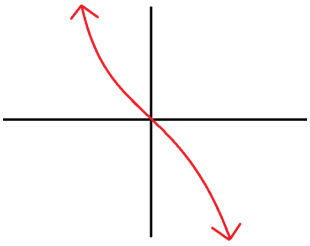
2) When n is even:

a) If the leading coefficient is positive ($a_n > 0$), the graph rises to the left and right.



b) If the leading coefficient is negative ($a_n < 0$), the graphs falls to the left and right.



Coefficient/Degree	Even Degree	Odd Degree
Leading Coefficient > 0		
Leading Coefficient < 0		

A table to summarize end behavior

2 Example

Graph $x^3 - 9x$:

First we will look at the leading coefficient and the degree. The leading coefficient is 1 (greater than 0). The degree is 3 (odd). Therefore, we can conclude that the graph of this function is going to fall to the left and rise to the right.

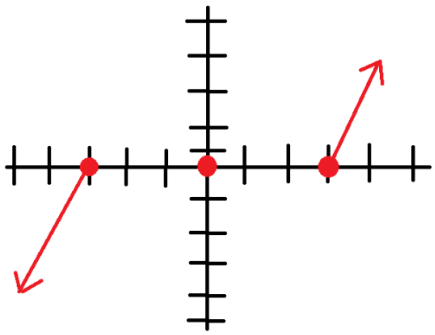
Next, we will find the zeros, or the points when the function $x^3 - 9x$ is equal to 0. We will do so by factoring:

$$x^3 - 9x = 0$$

$$x(x^2 - 9) = 0$$

$$x(x - 3)(x + 3) = 0$$

So our zeros will be $x = 0$, $x = 3$, and $x = -3$. We will begin our graph with the information that we have:

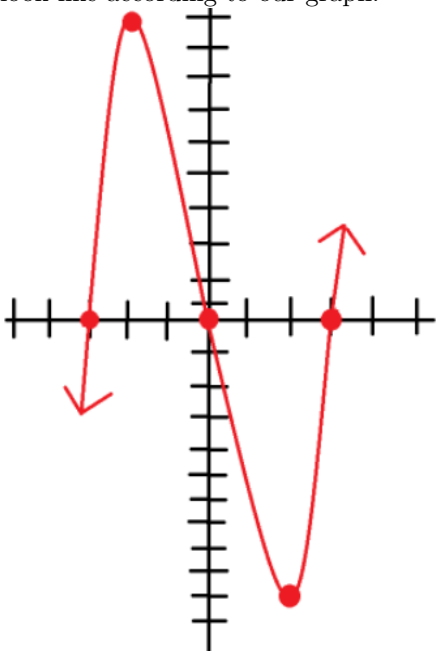


Next we will plug in some values of x into our equation. To get an idea of where the graph goes between -3 and 3, we will plug in -2 and 2:

$$(-2)((-2) - 3)((-2) + 3) = (-2)(-5)(1) = 10$$

$$(2)(2 - 3)(2 + 3) = (2)(-1)(5) = -10$$

So, when $x = -2$, $y = 10$, and when $x = 2$, $y = -10$. We will draw what this may look like according to our graph.



Thus, we have graphed $x^3 - 9x$.